Example LaTeXDocument

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1 Polynomials

A polynomial is a function that involves only addition, multiplication, and positive integer powers of the variables. For example, the equation for a line, $y = mx + b$, is a polynomial, as is the equation for a quadratic, $y = ax^2 + bx + x$. In general, a polynomial could be a function of multiple variables, $f \rightarrow f(x, y) = ax + by + cxy + d$, but in Physics, polynomials of a single variable are most common, i.e. a power series.

Any polynomial of a single variable x can be written as the sum of powers of x multiplied by coefficients.

$$f(x) = \sum_{i=0}^{n} a_n x^n \tag{1}$$

Here, n is the degree of the polynomial.

2 Integrals and Derivatives

The derivative of a function $f(x)$ gives the rate-of-change of the function with respect to x. In general, the derivative will also be a function of x:

$$g(x) = \frac{\mathrm{d}f}{\mathrm{d}x} \tag{2}$$

The Fundamental Theorem of Calculus relates an anti-derivative (integral) to the derivative. The first part states that, if

$$F(x) = \int_a^x f(t)\mathrm{d}t \tag{3}$$

then

$$\frac{\mathrm{d}F(x)}{\mathrm{d}x} = f(x). \tag{4}$$

Or, put another way

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\int_a^x f(t)\mathrm{d}t\right] = f(x) \tag{5}$$

The second part says that the value of a definite integral over a closed interval $[a, b]$ can be computed by taking the difference of the anti-derivative function at the end points of the interval.

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a) \tag{6}$$

where $F(x)$ is function defined in Equations 3 and 4

3 Partial Derivatives

A *partial* derivative is the derivative of a function of two-or-more variables with respect to one variable, while holding the other variables constant. For example, if we have $f \to f(x, y) = xy^2$, then

$$\frac{\partial f}{\partial x} = y^2 \tag{7}$$

because we treat y as a constant while taking the derivative. Similarly,

$$\frac{\partial f}{\partial y} = 2xy. \tag{8}$$

4 Units and Physical Constants

Physical quantities have dimensions and when we measure these quantities we express them as a numerical value with a *unit*. For example, we can measure a length with a ruler to get $L = 12.5\,\mathrm{cm}$.

That means that the value of the physical constants (for example G), will depend on the units we express the constant in. Here are some common physical constants expressed in their most common units.

Gravitational Constant:

$$\begin{aligned} G &= 6.674 \times 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\,s}^2} \\ &= 6.674 \times 10^{-11}\,\frac{\mathrm{N\,m}^2}{\mathrm{kg}^2} \end{aligned}$$

Boltzmann's Constant:

$$\begin{aligned} k &= 1.38 \times 10^{-23}\,\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}^2\,\mathrm{K}} \\ &= 1.38 \times 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}} \\ &= 8.617 \times 10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}} \end{aligned}$$

Planck Constant:

$$\begin{aligned} h &= 6.63 \times 10^{-34}\,\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}} \\ &= 6.63 \times 10^{-34}\,\mathrm{J\,s} \\ &= 4.136 \times 10^{-15}\,\mathrm{eV\,s} \end{aligned}$$

Reduced Planck Constant:

$$\begin{aligned} \hbar &= 1.054 \times 10^{-34}\,\frac{\mathrm{m}^2\,\mathrm{kg}}{\mathrm{s}} \\ &= 1.054 \times 10^{-34}\,\mathrm{J\,s} \\ &= 6.58 \times 10^{-16}\,\mathrm{eV\,s} \end{aligned}$$

5 Trig Functions

Figure 1 is a plot of three of the most common trig functions used in physics. The derivative and integral of many trig functions can be written in terms of other trig functions. Table 1 lists a few of these.

Function	Derivative	Integral
$\sin(x)$	$\cos(x)$	$-\cos(x) + C$
$\cos(x)$	$-\sin(x)$	$\sin(x) + C$
$\tan(x)$	$\sec^2(x)$	$\ln\lvert\sec(x)\rvert + C$

Table 1: List of the derivative of some trig functions.

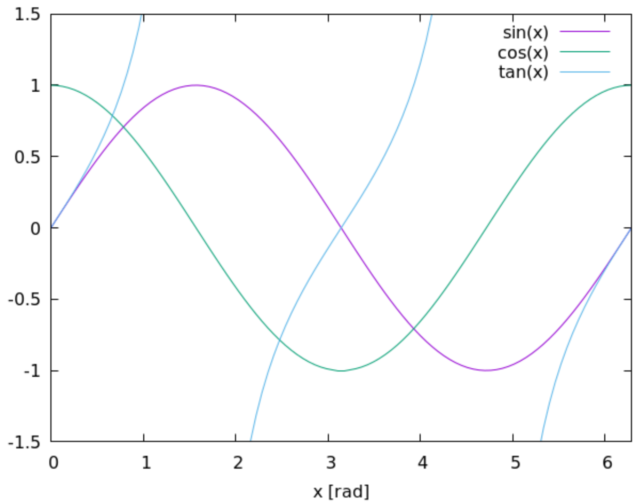

Figure 1: The three trig function: $y = \sin(x)$, $y = \cos(x)$, and $y = \tan(x)$.