Homework 06

For problems that require you to write a script, save your script in a directory named `scripts/` in the top of the homework directory. Write your script assuming the data file(s) it will use are in the current working directory. I will run your script by first `cd`'ing to the directory containing the data file to be plotted, opening `gnuplot`, and loading it with the `load` command.

1. The homework directory contains a file named `data/KC-trip.txt`. This file contains data that was collected on a trip from Hays to Kansas City. Every thirty minutes, I wrote down the distance I had travelled, since leaving Hays. The first column is time, in minutes, the second column is distance, in miles. Knowing that I was in Hays when I started the clock, fit a line to the data to determine how fast I was driving, on average, in miles per hour. Then create a graph of the data with the best fit line that looks like the graph in Figure 4. Name your script `KC-trip.gnuplot` and have your script create a png named `Figure4.png`.
2. The homework directory contains a file named `data/damped_harmonic_oscillator.txt`. This data was collected while observing a mass on spring in room 205. The data was collected with a program that used a video camera to watch the mass and automatically log its position. The first column is time (in seconds), the second column is displacement (in centimeter).
   The system is an example of a damped harmonic oscillator. A damped harmonic oscillator is an oscillator that has a frictional force acting on it so that it will eventually come to rest. The position of a damped harmonic oscillator as a function of time is given by $x(t) = x(0)e^{-t/\tau}\cos(\omega t)$, where τ is the time constant of the frictional decay, ω is the angular frequency of oscillation, and $x(0)$ is the oscillator's initial position.
   1. Fit a line to the data and determine the time constant for frictional decay.
   2. Fit a line to the data and determine the angular frequency of the oscillator.
   3. Fit a line to the data and determine the period of oscillation.
   4. How fast was the mass moving at $t = 2\,\mathrm{s}$? Recall that the velocity of an object is given by the time derivative of its position.
   5. What maximum speed did the mass reach while being observed?
3. The homework directory contains a file named `data/kinematics.txt`. This data was collected while observing a $1\,\mathrm{kg}$ mass move along a frictionless track under constant acceleration. The first column contains the mass's position (in centimeters) and the second column contains the time at which it reached the position.
   1. Recall that under constant acceleration, the position of the object will be given by the kinematics equation
      $$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$
      Fit this kinematics equation to the data contained in the file to obtain values for x_0, v_0, and a.

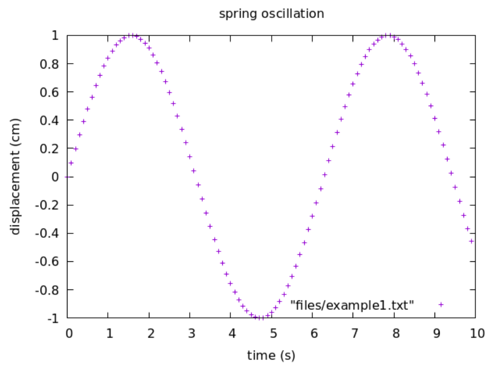

Figure 1:

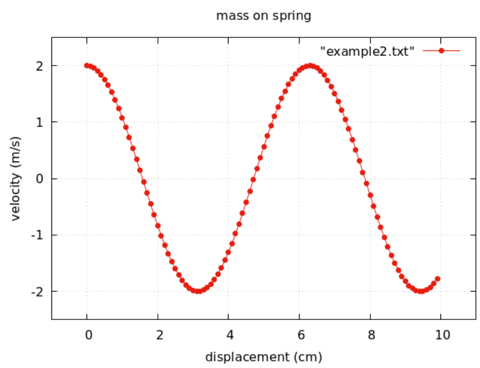

Figure 2:

2. Based on your fit, what was the maximum position reached by the mass during the time that it was observed?

3. The position of an object at some time t can always be written as

$$x(t) = x_0 + \bar{v}t$$

where $\bar{v}$ is the average velocity of the object over the time t. The average velocity can be calculated from position:

$$\bar{v} = \frac{x(t) - x_0}{t}$$

For constant acceleration, the average velocity of an object is just

$$\bar{v} = \frac{v_0 + v}{2} = v_0 + \frac{1}{2}at$$

Write a gnuplot script named `kinematics-avg_vel.gnuplot` that will create a png named `kinematics-avg_vel.png` with a plot of the average velocity vs. time using the data from the file (you will need a value for x_0 to do this, use the value you obtained from your first fit). Then fit a function to this data and include it on your plot. You can determine values for v_0 and a from your fit, check that they are consistent with the value you obtained from the first fit.

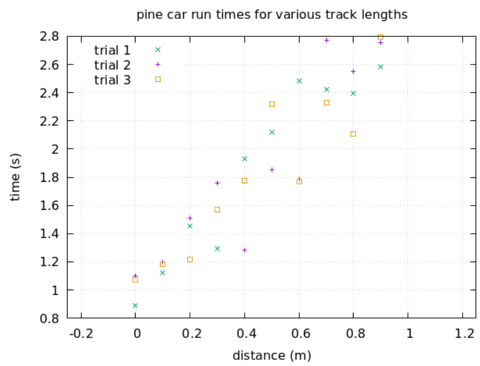

Figure 3:

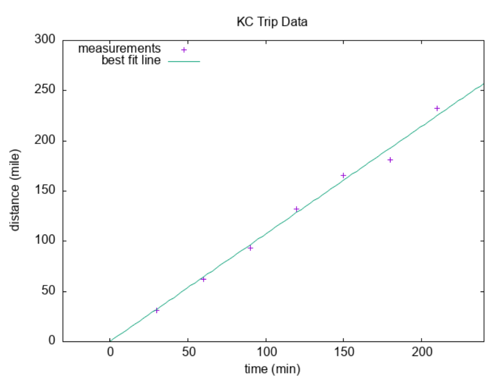

Figure 4: